~~• Notation~~

(I) • Duality ⇔ • ST-moves → ~~• Kirby moves,~~ ~~→ • brane webs~~

3d N=4

2310.07624

w/ Piotr. 2302.13371

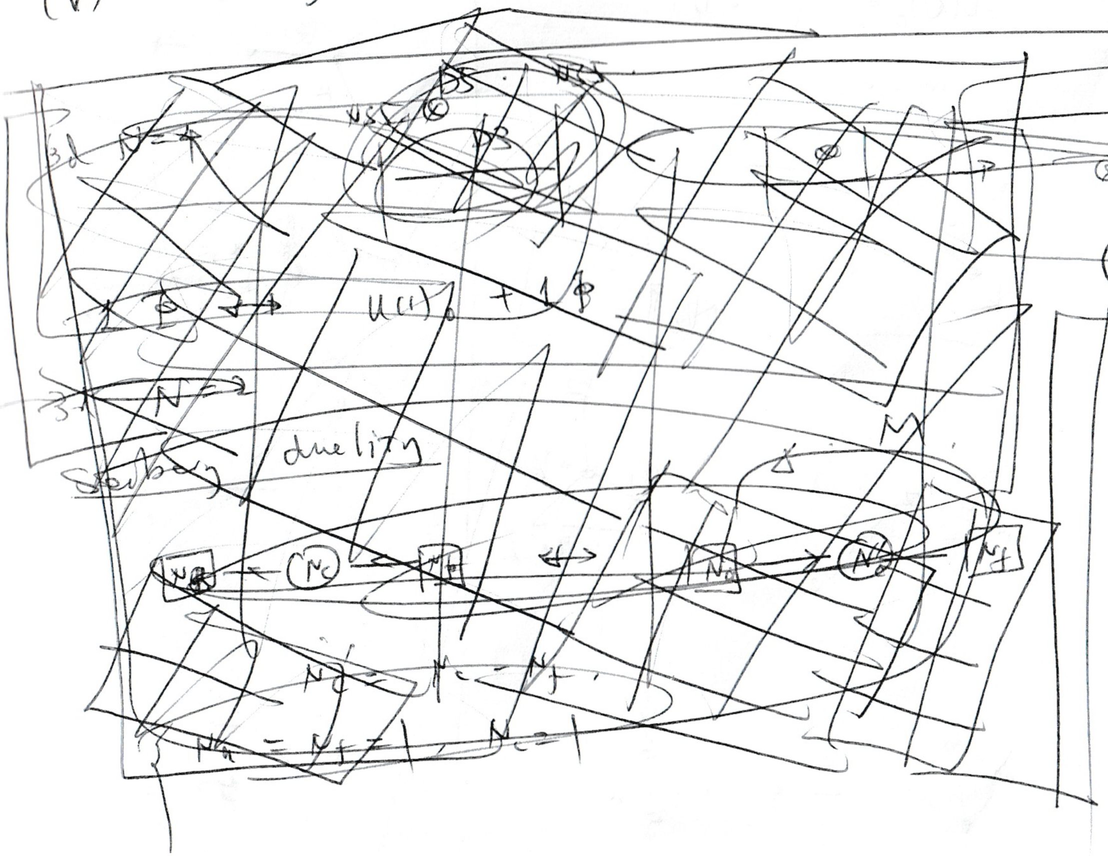
(II) • Three-manifold → Kirby move. $\otimes - \alpha$, $- \beta$

(III) ~~• 3d brane web → defect M5-brane →~~

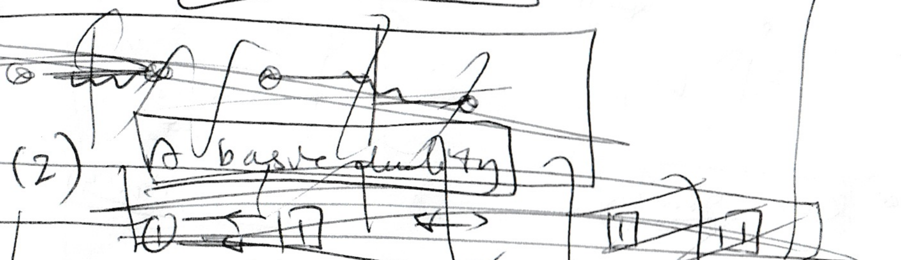
(IV) • ~~defect M5 →~~ Lens space → defect M5-brane → 3d - brane webs, ~~charges~~ charges

(V) • Plumbing three-manifold → derive ST-moves,

Surgery construction for 3d theory



~~Gauge charges~~



(2) ~~abelian duality~~

3d N=4 $U(1)_0 + 1(Q, \tilde{Q}) + 1$ adj ↔ $(q, \tilde{q})$

$W = \Phi_a Q \tilde{Q}$, $W = 0$.

SQED ↔ XYZ,

$U(1)_0 + 1 Q + 1 \tilde{Q}$ ↔ X, Y Z

$W=0$. $W = XYZ$,

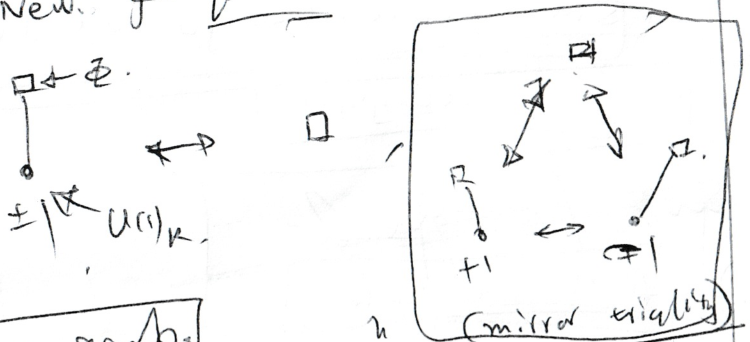
• two Φ are decoupled,

$U(1) + 1\,Q \leftrightarrow 1\,\tilde{Q}\,Q$

$k^{eff} = \pm 1$

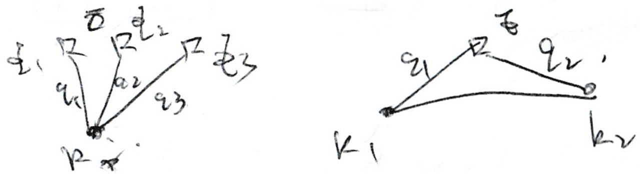
• Not convenient to see conducts. quiver diagram

• New quiver

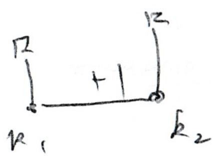


• Plumbing graphs

$$k^{\Phi}_{eff} = k^{\Phi}_{bare} + \sum_{i=1}^{N_f} q_i^2 \frac{\mathrm{sign}[q_i]\,\mathrm{sign}[m_i]}{2}$$



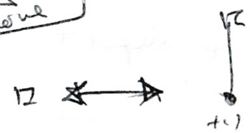
mixed CS levels



$U(1)_{k_1} \times U(1)_{k_2} + 2\,\Phi$

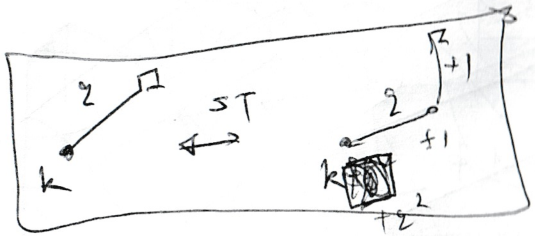
• benefit: represent charges & CS k_{ij}

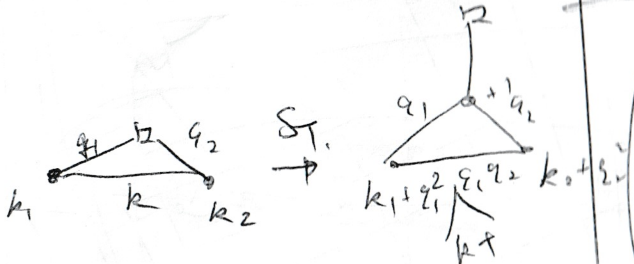
ST-move



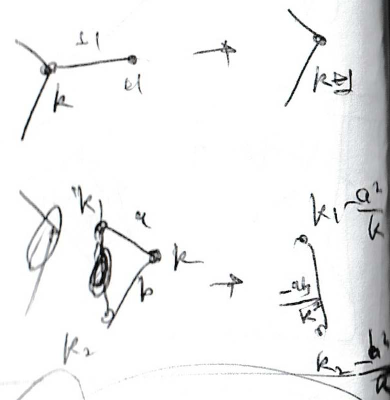
$U(1)_F \leftrightarrow U(1)_T$

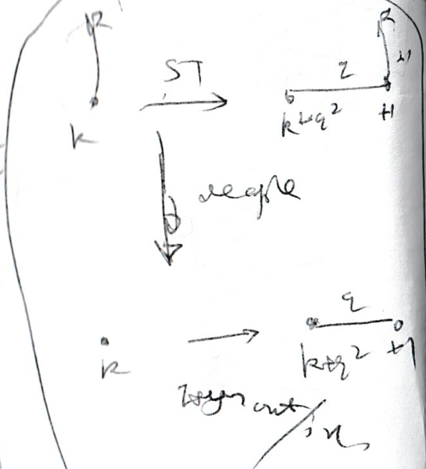
$m \leftrightarrow \xi$





• Integrate out/in gauge groups,





~~Three mfd~~ ! Basic 3-mfd Lens spaces.

• ex : S^3, $S^1 \times S^2$.

$L(1, n) = S^3$, for $n \in \mathbb{Z}$,

$L(k,1)$, $S^3/\mathbb{Z}_k$,

$L(1,1) = S^3$

$L(0,1) = S^1 \times S^2$

$|z_1|^2 + |z_2|^2 = r^2$,

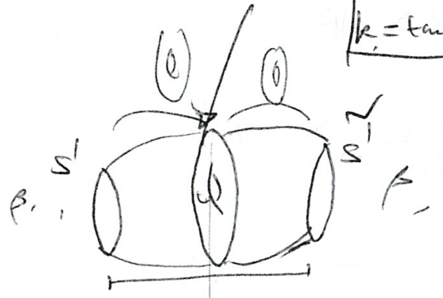
$(z_1, z_2) \mapsto (z_1 e^{\frac{2\pi i}{k}}, z_2 e^{\frac{2\pi i}{k}})$,

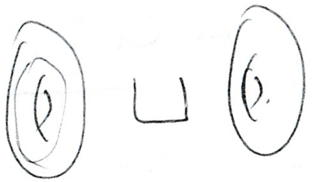
gluing map. $SL(2,\mathbb{Z})$

$k = \tan\theta$

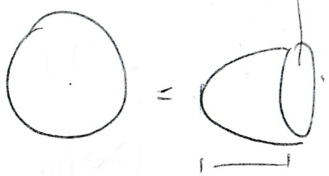
• How to construct 3-mfd.

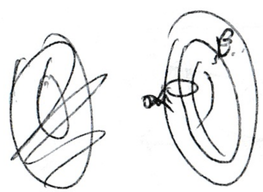
• How to understand Lens space.



$L(k,1) =$ 

$\partial(D^2 \times S^1) = T^2$

$D^2 =$  $= S^1 \downarrow \mathbb{R}_+$



(α, β),

$L(k,1)$: $T^2 \hookrightarrow L(k,1)$
$\downarrow \pi$
I.

• ~~Application~~ How to use it ?

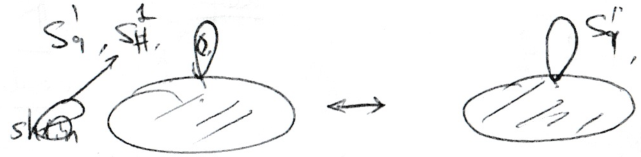
M-th, M5, 6d (2,0).
6d

M5/M3 $= \mathbb{R}^3$

$\to$ 3d $T[M3]$
Abelian theory

1 M5 / $L(k,1)$ = ?

M-theory / IIB duality.



M-th
↓ shrink $S^1_\#$

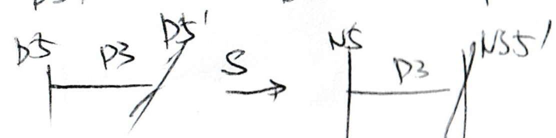
IIA ($S^1_{q_\alpha}$) $\xleftrightarrow[x_q]{\text{T-dual}}$ IIB ($S^1_{q_\beta}$).

M5 (9#) ~~M5~~ D0 .. (# $S^1_\#$)

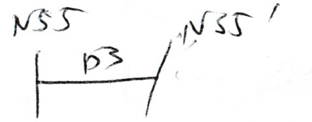
↓ ↓ KK

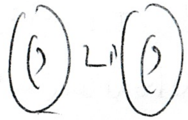
D4 ⟷ D3. D6 ⟶ D5

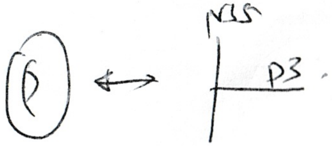
M5 / $L(k,1)$.



~~3d Σ~~ 3d $T[L(k,1)] = U(1)_k$

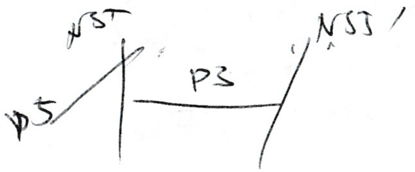






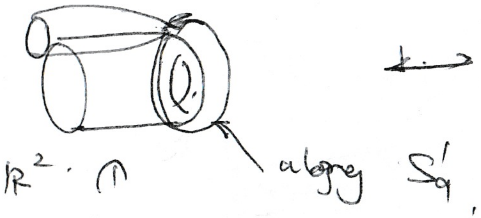
? $U(1)_k + 1\,\Phi$.

- ~~When~~ When/How to couple the matter?
- Hint: 3d brane web



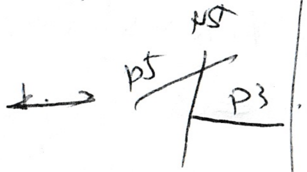
$D3 \to \Phi$.

- There is another way to introduce D5-brane?



$\mathbb{R}^2 \cap$

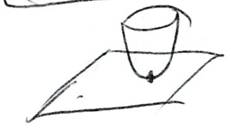
$T^*[L(k,1)]$

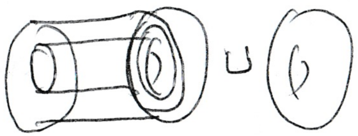


$T^* M_3 = M_3 \times \mathbb{R}^3$.

defect M5: $\boxed{\mathbb{R}^2 \times_q S^1_q} \times \mathbb{R}^3_{3d}$, codim 2.

$$\boxed{(\text{defect M5}) \cap L(k,1) = S^1_q}$$

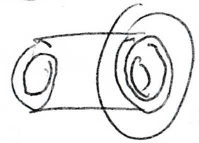




$U(1)_k + 2\,\Phi$.

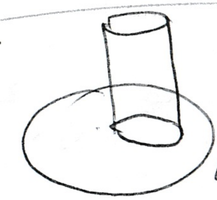
winding # = charge

ex:



Φ ~~q = 2~~

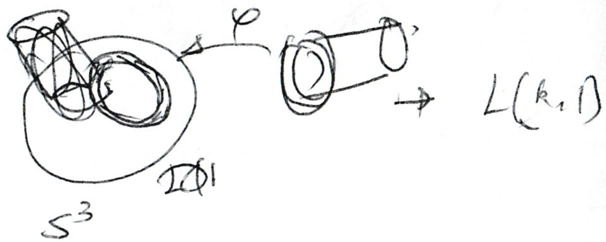
$q = 2, -2$



Ooguri–Vafa defect in knot theory in 3d CS theory, $\langle O_W \rangle$, Wilson loop

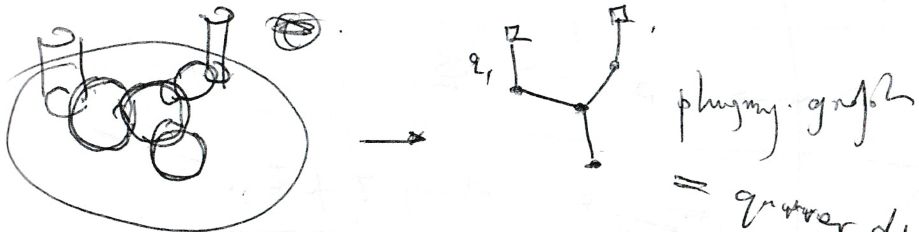
- How about general 3-manifold

Dehn surgery



• Lickorish – Wallace

any ~~complete~~ compact connected 3-mfd obtains [illegible]



plumbing graph = quiver diagram

$M3 = L(k_1;1) \sqcup L(k_2,1) \cup L(k_n,1) \ldots$

↓ building block.

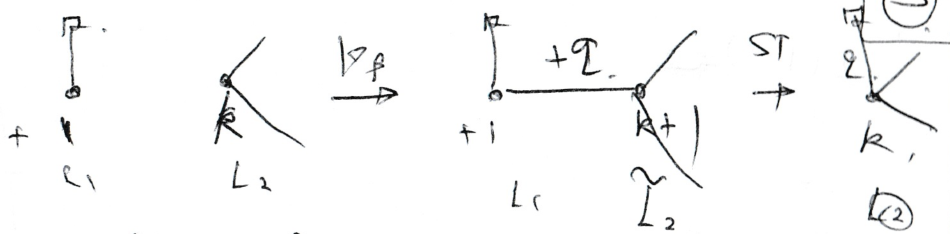
• $M2 \cong M3$

Kirby-move ~~α, I, II~~,

$-\alpha, -\beta$

Kβ : handle-~~slides~~

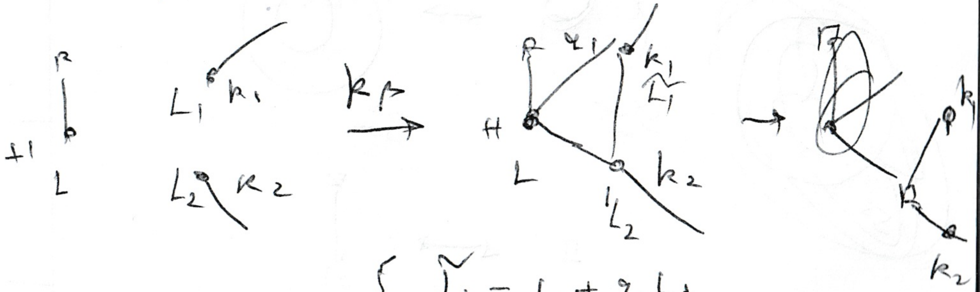
$\tilde{L_2} = L_2 \pm L_1$



$\tilde{L}_2 = L_2 + q L_1$

q = winding number

• bi-fold move



$$\begin{cases} \tilde{L}_1 = L_1 + q_1 L_2 \\ \tilde{L}_2 = L_2 + q_2 L_1 \end{cases}$$

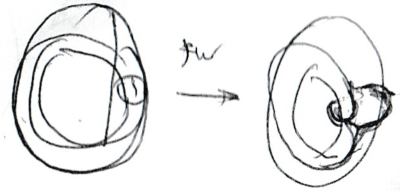
Kβ : intertwine matter

• GL-fn doesn't depend on q_i, change q_i

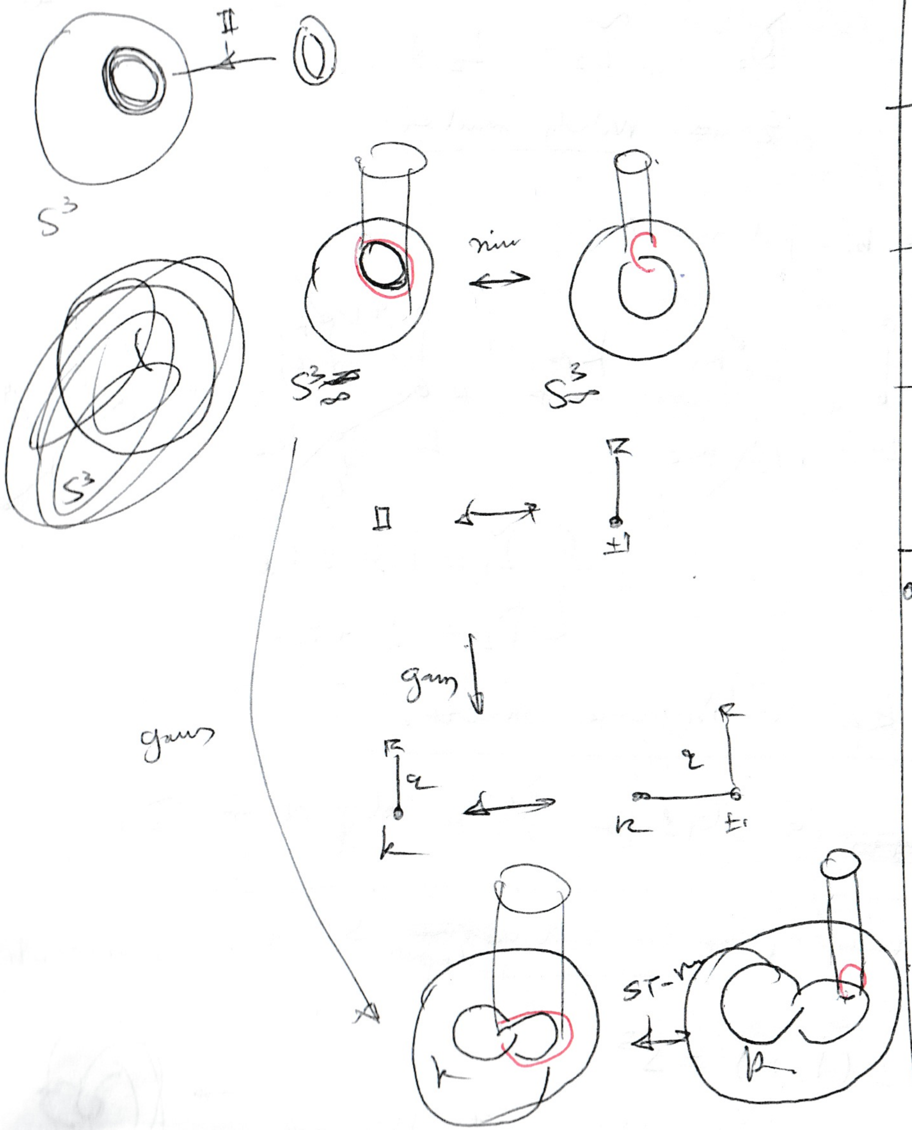
• ~~what~~ We have not ~~[illegible]~~ interpret ST-move geometrically

$L(1,n) = S^3$,

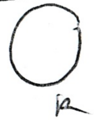
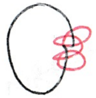
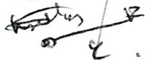
Dehn twist



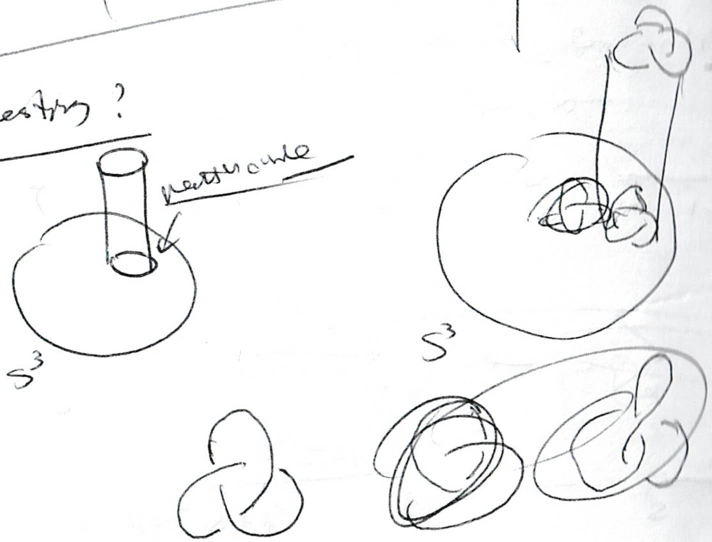
$L(1,0) \cong S^3_\infty$



Dictionary?

 K	↔ • K ↔	$U(1)_k$
	↔ ? →	$\mathbb{Z}$
	↔  q	charge-q of $\mathbb{Z}$
K_α	↔	conjugate w/ ~~...~~
K_β	↔	add $\mathbb{Z}$

∘ Open questions?



? $3d\ T[S^3 \setminus K] = ?$

K_{4_1}