

# Surgery construction for 3d theories

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Ref: SC 2310.07624, SC & P. Sułkowski 2302.13371

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## Introduction

The 3d  $N=2$  gauge theories are expected to be geometrically engineered by 3-manifolds. Inspired by DGG's work, we propose a Dehn surgery construction, after taking into account both physical aspects and various properties of 3-manifolds.

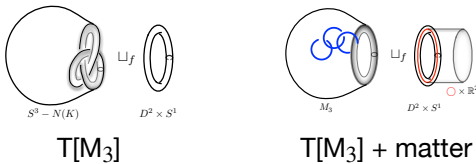
## Motivation

In 2302.13371, from physical perspectives we found that various 3d abelian dualities can be generalized by gauging global symmetries, in which matter fields and mixed Chern-Simons levels are compulsory. To describe various dualities, we proposed a special type of quiver diagrams, which match with plumbing graphs of 3-manifolds, but is a generalized version due to the presence of chiral multiplets.

However, 3-manifolds themselves cannot give matters. This motivates us to think about the corresponding geometric objects for matters.

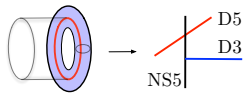
## Result

In 2310.07624, we found that matters are engineered by Ooguri-Vafa(OV) defects, which was originally introduced to give Wilson loops in Chern-Simons theories. But in our setting, OV defects can introduce chiral multiplets. Dehn surgery construction is as follows. First, one wraps a single M5-brane on the bulk 3-manifolds to get a pure gauge theory  $T[M_3]$ . Note that the knot for  $M_3$  can be transformed into links by Kirby moves. Second, a M5-brane wraps on external non-compact Lagrangian 3-manifolds (OV defect) on the tangent bundle of  $M_3$  to engineer matters.



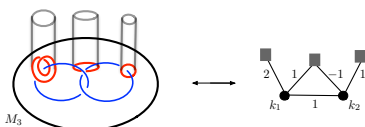
## Evidence-I

OV defect is dual to a D5-brane of 3d brane webs in type IIB string theory. Some 3d brane webs are already known to be given by D3-NS5-D5. We note that surgery construction can even suggest new brane webs.



## Example

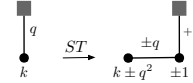
One can introduce many OV defects on a 3-manifold given by Dehn surgery along links. For instance, a theory with two  $U(1)_k$  gauge groups and three chiral multiplets with various charges can be represented as



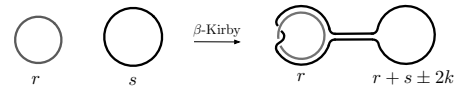
where the right graph is the plumbing graph. The black nodes denote gauge groups, and gray boxes denote matters. Numbers are charges or CS levels.

## Kirby moves and rational equivalent surgery

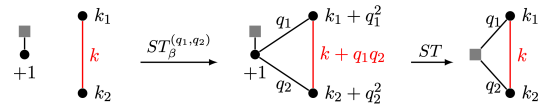
To find more evidence, we should compare geometry and physics. To begin with, 3-manifolds have equivalent surgeries, such as Kirby moves of  $\alpha$ - and  $\beta$ -types. Kirby- $\alpha$  is shown in literature as integrating out/in gauge fields, which can be promoted as ST-moves (gauged mirror duality) in the presence of matter.



Kirby- $\beta$  is known as handle slides, which are the recombinations of circles for Dehn surgery.

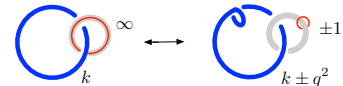


which can be promoted to a geometric operation (not a duality) of introducing an OV defect.



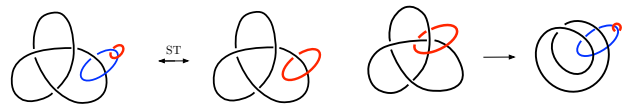
This Kirby- $\beta$  along with ST-moves suggest that a matter (OV defect) should correspond to a circle with framing number- $\infty$ , which is the intersection of  $M_3$  and OV defect, and we call it matter circle, and denote it by  $\bigcirc_\infty$ .

On the neighborhood of  $\infty$ -circle, there is an identical Dehn surgery, and what follows is that rational equivalent surgery found by Rolfsen can be applied to turn it into a Lens space  $S^3_{\pm 1}$ . Applying these in turn we find a more deep geometric interpretation for ST-moves. More details are in 2310.07624.



## Application

Matter circles as elements in the fundamental group  $\pi_1(S^3 \setminus K)$ , enable one to intuitively read off gauge theories and manipulate 3-manifolds.



## Dictionary, evidence-II

In sum, physical dualities are interpreted as various geometric transformations. Structures of 3-manifolds perfectly agree with physical aspects of 3d gauge theories.

three-manifolds	plumbing graphs	abelian gauge theories
matter circle $\bigcirc_\infty$	$\blacksquare$	chiral multiplet $\mathbf{F}$
gauge circle $\bigcirc$	$\bullet_{k_i}$	gauge group $U(1)_{k_i}$
winding num. between $\bigcirc$ and $\bigcirc$	$\bullet_{q_i}$	charges $q_i$ of fund. rep.
linking num. between $\bigcirc$ and $\bigcirc$	$\bullet_{k_{ij}}$	effective CS levels $k_{ij}$
$\alpha$ -Kirby moves on $\bigcirc$	blow up/down of $\bullet_k$	integrate in/out $U(1)_k$
rational equivalent surgery (4.25)	ST-moves (4.26)	gauged mirror triality
handle-slides of $\bigcirc$ (4.29)	add $\blacksquare$	add chiral multiplets

## Open problems

Matter knots and superpotentials.